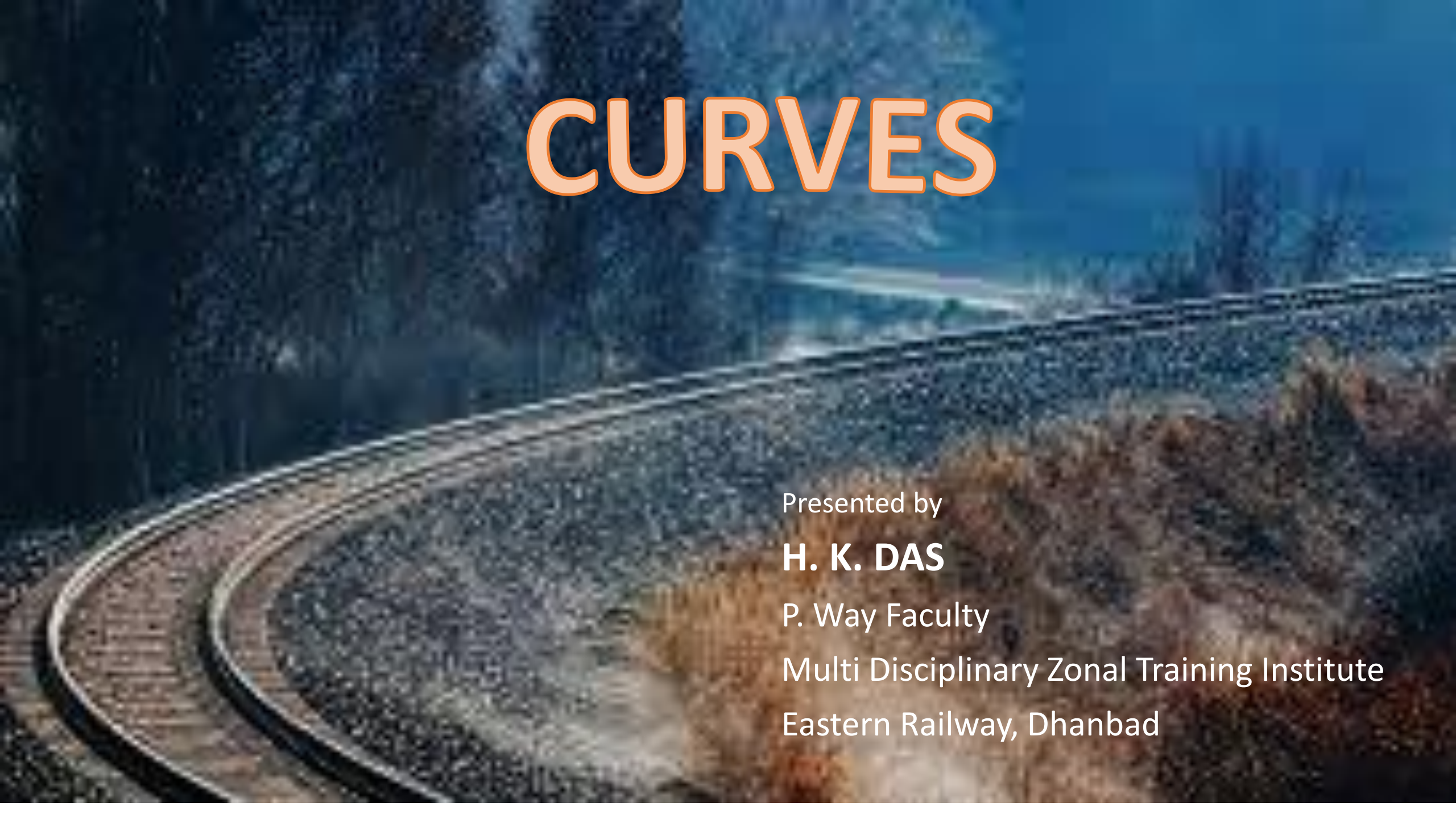

CURVES

Presented by

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Introduction

- For railroad, preferred alignment is straight.
- When straight alignment cannot be continued, a curve is necessary.

- **Necessity of curves**

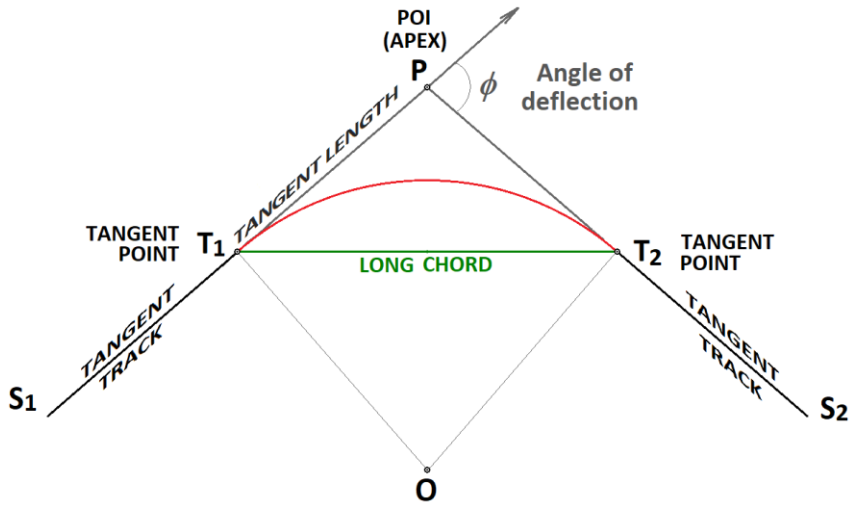
➢To bypass obstructions

➢To pass through obligatory or desired points

➢To provide extra length for regulating gradient

- Curve is introduced between two given straights tangentially. The two straights are called **tangent tracks**.
- Railway curves are **circular**, with or without transitions.
- The radius of the **outer** rail is reckoned as the radius of the curve.

ELEMENTS OF A SIMPLE CIRCULAR CURVE

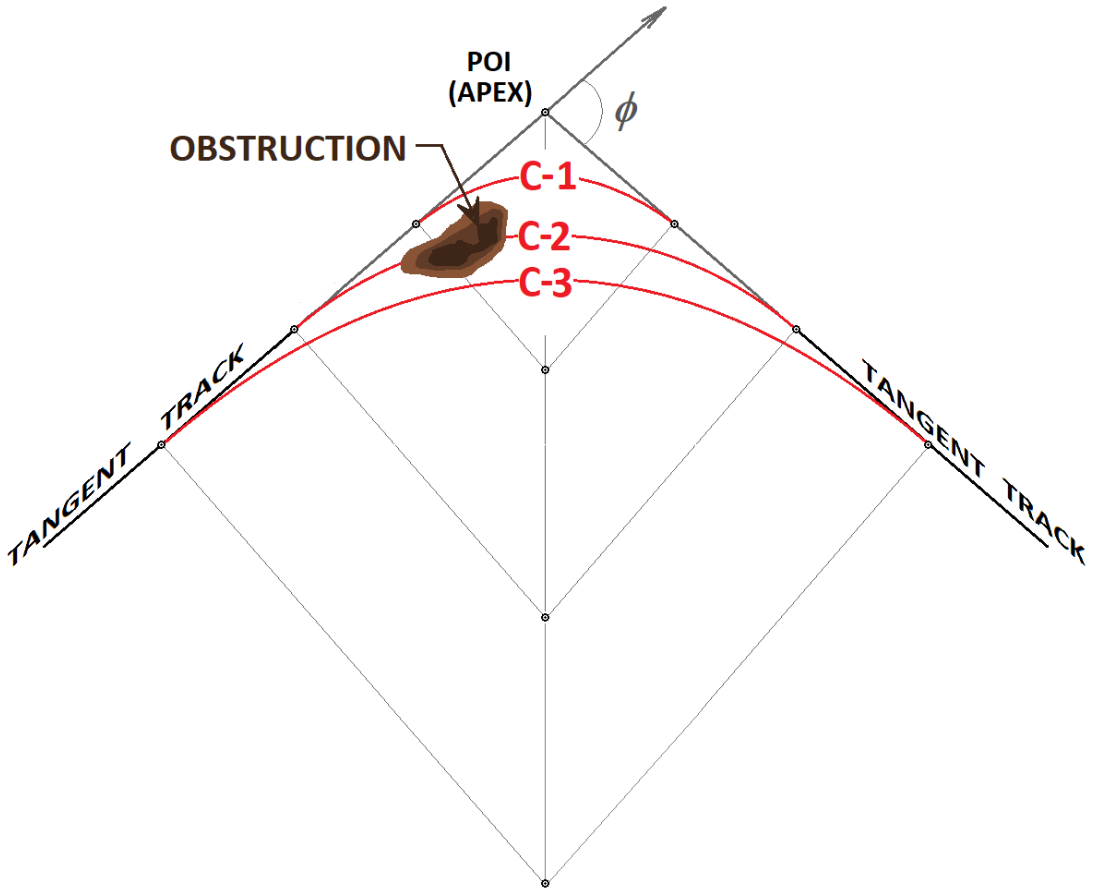


SIMPLE CIRCULAR CURVE

- Tangent tracks, $\mathbf{S_1T_1}$ & $\mathbf{T_2S_2}$
- Point of Intersection (of tangents), POI or Apex, **P**
- Angle of Deflection (of tangents), **ϕ**
- Radius, **R**
- Tangent Length, $\boldsymbol{T_1P=PT_2= T\ (say)}$

$$\boldsymbol{T = R\tan\frac{\emptyset}{2}}$$

- Tangent Points, $\mathbf{T_1}$ & $\mathbf{T_2}$
- Long Chord, $\mathbf{T_1T_2} = \boldsymbol{2R\ sin\frac{\emptyset}{2}}$
- For given tangent tracks, **ϕ** is fixed.

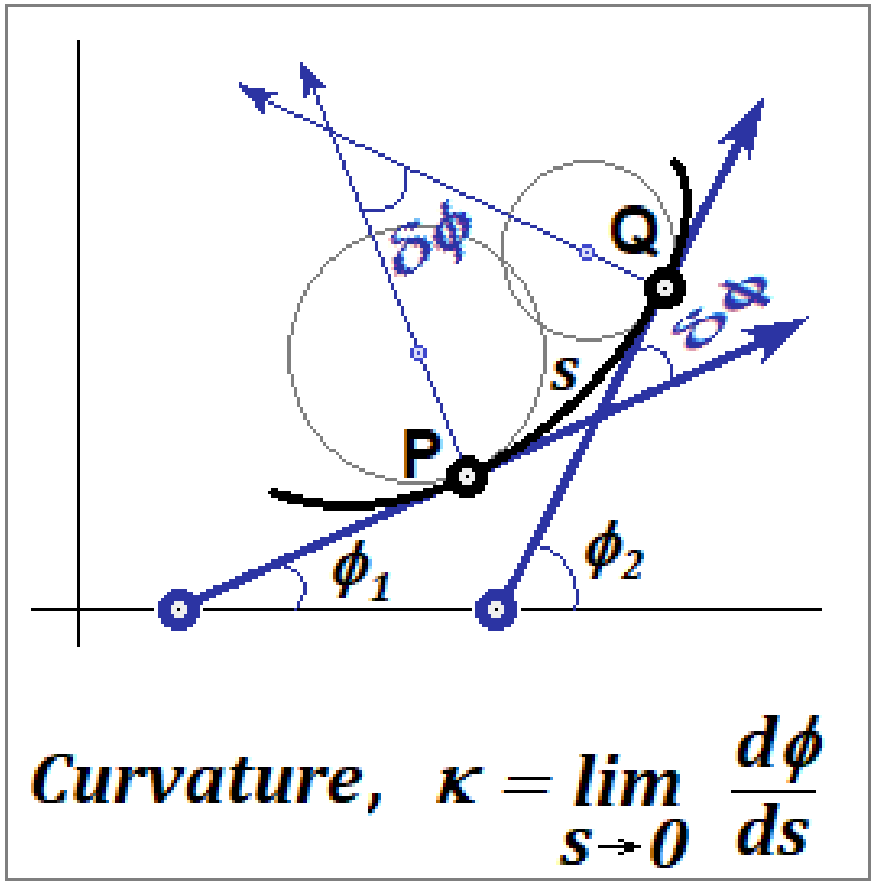


- Between two given tangent tracks, infinite numbers of curves can be fit with different radii and different tangent point locations.
- Of these numerous options, optimum alignment is choosen.
- AS per IRSOD, sharpest permissible radius of BG curve is 218 m (8°)
- Narrower gauges permit sharper radii: 109 m (16°) for MG; 44 m(40°) for NG.

TOPICS, THIS SESSION

- Curvature
- Degree of a curve
- Versine
- Cant or Superelevation
- Equllibrium cant and equilibrium speed
- Cant Deficiency and Cant Excess
- Transition Curve

CURVATURE



- Curvature means the quality of being curved or the amount of bending.
- It is the instantaneous rate of change of direction of a point that moves on the curve.

$$\kappa = \lim_{s \to 0} \frac{d\emptyset}{ds}$$

- Curvature of a curve at a point P on it is the reciprocal of the radius of the *osculating circle* tangential at P.

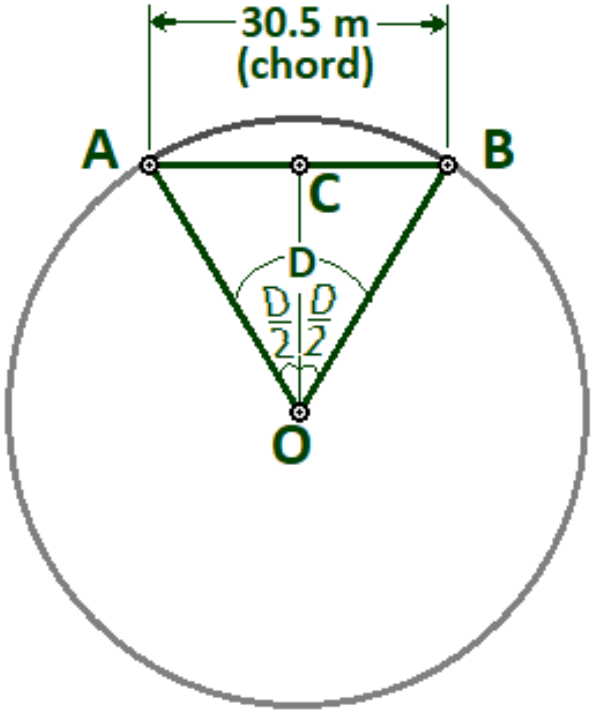
$$\kappa(P) = \frac{1}{R(P)}$$

- Curvature of a straight is zero.
- The curvature of Railway curves is expressed in terms of *Degree of the curve*.

DEGREE OF A CURVE

Degree of a curve is the angle in degree subtended by a 30.5 m chord at its centre.

It is a measure of curvature of a circular curve.



$\triangle OAC$: AC=15.25 m, OA= Radius, R

$$\sin \angle AOB = \frac{AC}{OA}$$

$$\frac{D}{2} = \angle AOB = \sin^{-1} \frac{15.25}{R}$$

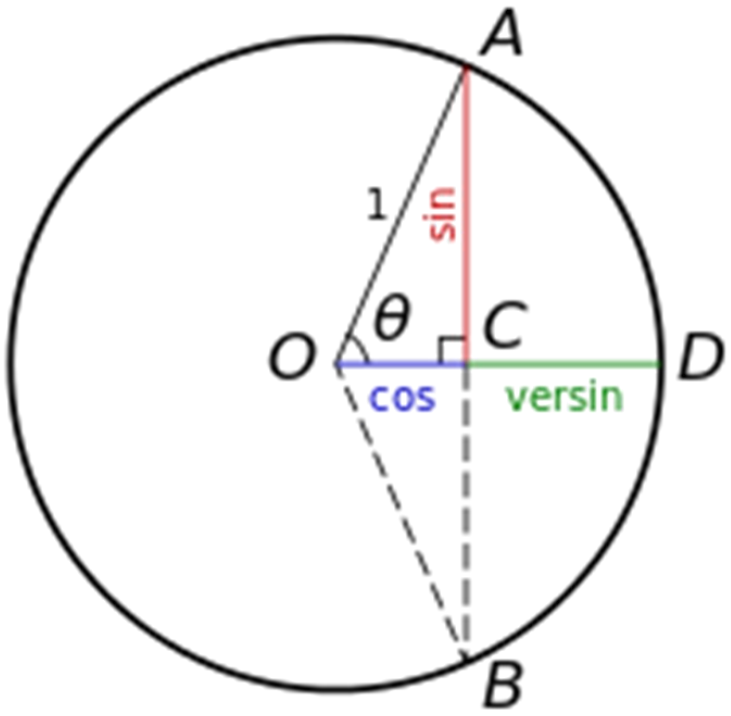
$$D = 2 \sin^{-1} \frac{15.25}{R} \approx \frac{2x15.25}{R} \text{ (in radian)}$$

$$\approx \frac{30.5}{R} x \frac{180}{\pi} \text{ degree}$$

$$\boldsymbol{D = \frac{1750}{R}} \text{ when R in m}$$

This means, a 1° curve deflects @ 1°/30.5 m.

VERSINE: Definition



- **As a trigonometric fuction:** Versine i.e. Versed Sine of an angle is **One minus Cosine** of that angle.

$$versin\,\theta = 1 - cos\theta = 2R\,sin^2\theta$$

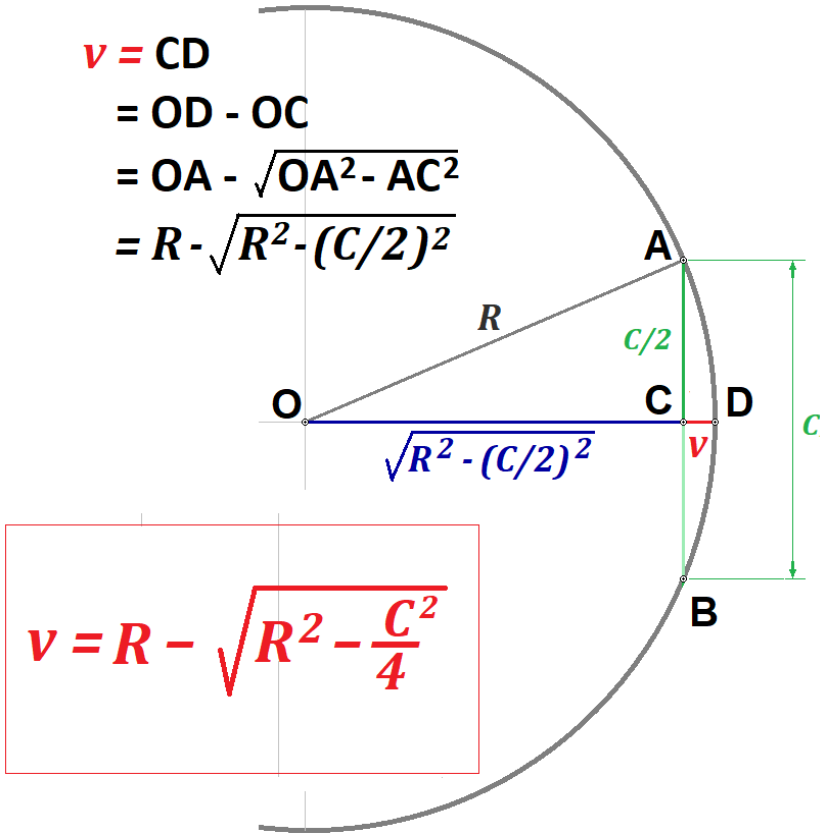
Note: For all θ, versine is always positive and ranges between 0 and 2.

- **As Chord Offset:** Versine is the distance between centre of chord and centre of arc.

Note: In some Railways, versine is referred to as **midchord offset.**

PicCredit: wikipedia

RELATION BETWEEN VERSINE, CHORD AND RADIUS



$$v = R - \sqrt{R^2 - \frac{C^2}{4}}$$

$$R - v = \sqrt{\left\{R^2 - \frac{C^2}{4}\right\}}$$

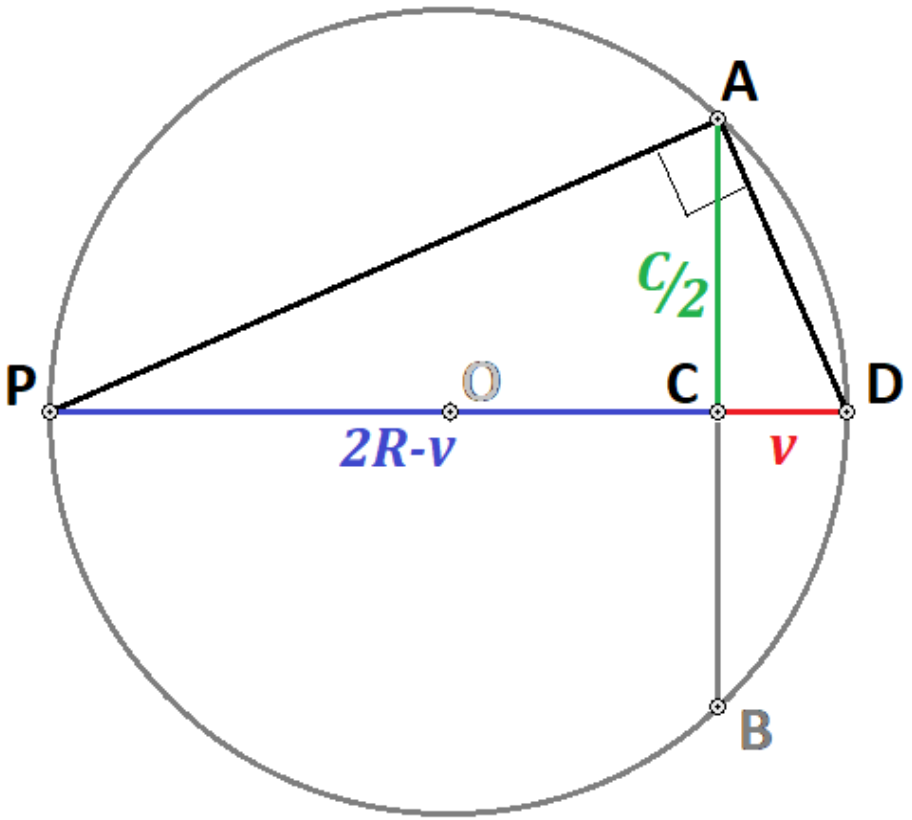
$$\Rightarrow \quad R^2 - 2Rv + v^2 = R^2 - \frac{c^2}{4}$$

$$\Rightarrow \quad R = \frac{C^2}{8v} + \frac{v}{2}$$

Since R is too large in the case of railway curves, and C is very small compared to R, $\frac{v}{2}$ being very small can be ignored.

$$\boldsymbol{v \approx \frac{C^2}{8R}}$$

Alternative Proof



Right angled triangle PAD

C is drop of A on hypotenyuse PD, the diameter of the circle.

CD is versine on chord AB=v

PC=2R-v

By Geometric Mean Theorem,

$PC.\ CD=AC^2$

$(2R-v).v=(C/2)^2$

$2Rv-v^2=C^2/4$

$$v=\frac{C^2}{8R}+\frac{v^2}{2R}$$

Since v is very small compared to R, we ignore v^2/R.

$$v=\frac{C^2}{8R}$$

Versine is widely used in Railway Engineering.

It is apllied in *Hallade method of track geometry survey* for checking the unevenness and alignment of track on a specified chord.

- In the limit as the chord C tends to zero, the ratio $\frac{\boldsymbol{C^2}}{\boldsymbol{8v}}$ tends to instantaneous radius.

$$\lim_{C \to 0} \left(\frac{C^2}{8v} \right) = R_i$$

IRPWM Para 401
Determination of Radius –

- **IRPWM Para 401 (1)**

The radius of a curve is determined by measuring the versine on a chord of known length, from the equation,

$$\boldsymbol{R = \frac{125\ C^2}{v}}$$

Where $R = Radius\ in\ metre$
$C = Chord\ length\ in\ metres, and$
$v = Versive\ in\ millimetres$

- **IRPWM Para 401 (2)**

Curves can be **designated** by radius in metres or by its degree.

Curves shall be **described** invariably by the radius in metres.

- $\boldsymbol{v \approx \frac{C^2}{8R}}$

when v, C & R are in same unit.

- $\boldsymbol{v \approx \frac{125\ C^2}{8R}}$

when v in mm, but C & R in m.

- $\boldsymbol{v \approx \frac{50000}{R}}$

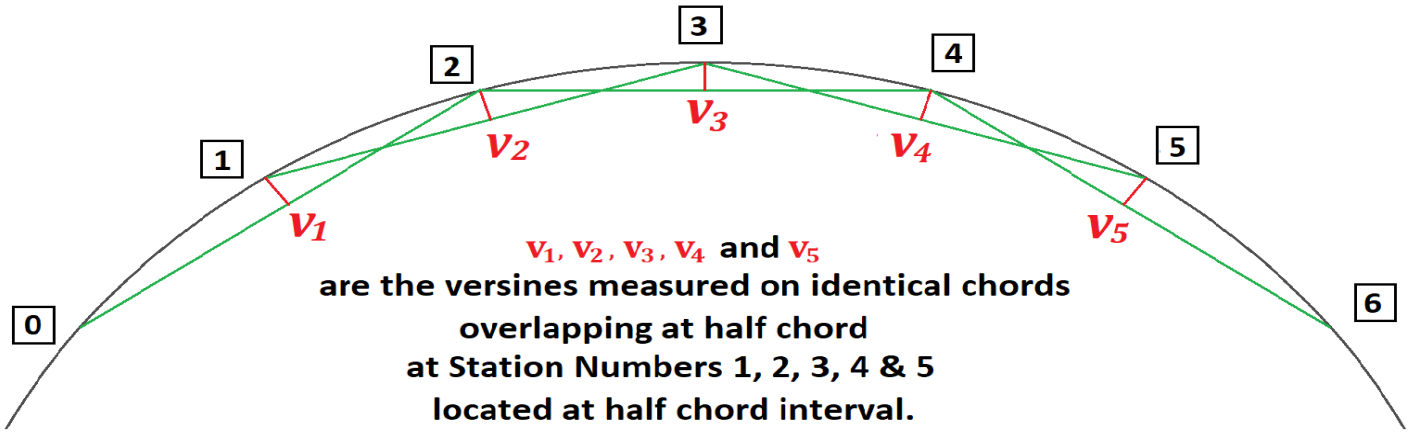
when versine in mm is measured on 20 m chord, R in m.

- $\boldsymbol{v = 28.57\ D}$

where versine in mm, D is Degree of the curve.

IRPWM Para 401(3)

For measuring versines of a curve, 20 metres overlapping chords should normally be used with stations at 10 metres interval.



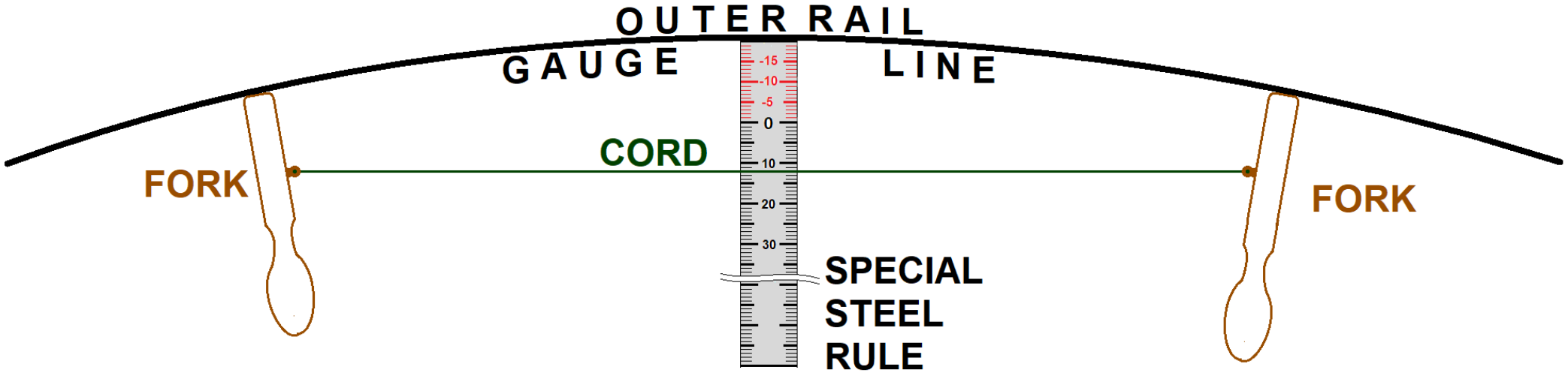
For checking the radii of the turnout and turn-in curves, overlapping curve of 6 metres should be used and the versine measuring stations should be located at every 1.5 metres.

(The turnout curves can also be checked by off-sets from the straight with the versine measuring stations 1.5 metres apart.)

Note: In IRPWM para 237(4)(b) &(c), it is stated to check versines at 3.0 m intervals “during maintenance”.

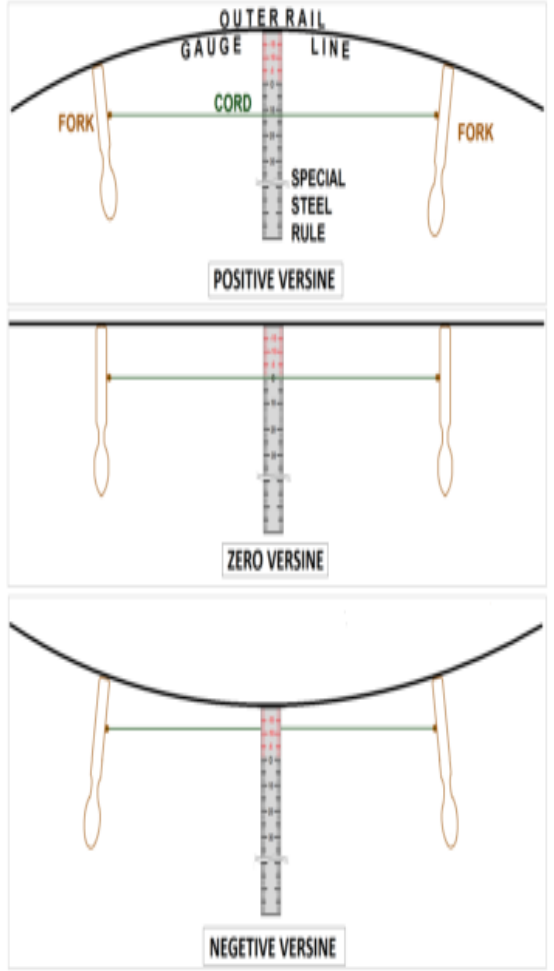
IRPWM Para 401(4)

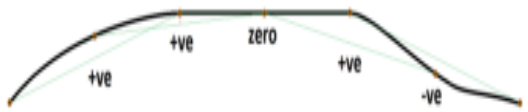
- The versine is obtained by stretching a fishing/nylon cord or wire taut between the end of chord length decided upon, and the measuring distance between cord or wire and gauge face of the rail at the middle point of the chord. Care should be taken that the cord or wire is applied to the side of the head of the rail at the gauge point.



Versine Measuring Gadget

- The versine measuring gadget consists of 2 Forks, a nylon chord tied to them at 20 mm from rail gauge face and 13 mm below rail top and a special measuring scale, the “zero”of which is fixed 20 mm away from rail.
- This allows the surveyor to read negetive versine, if any, without shifting the chord to other rail.



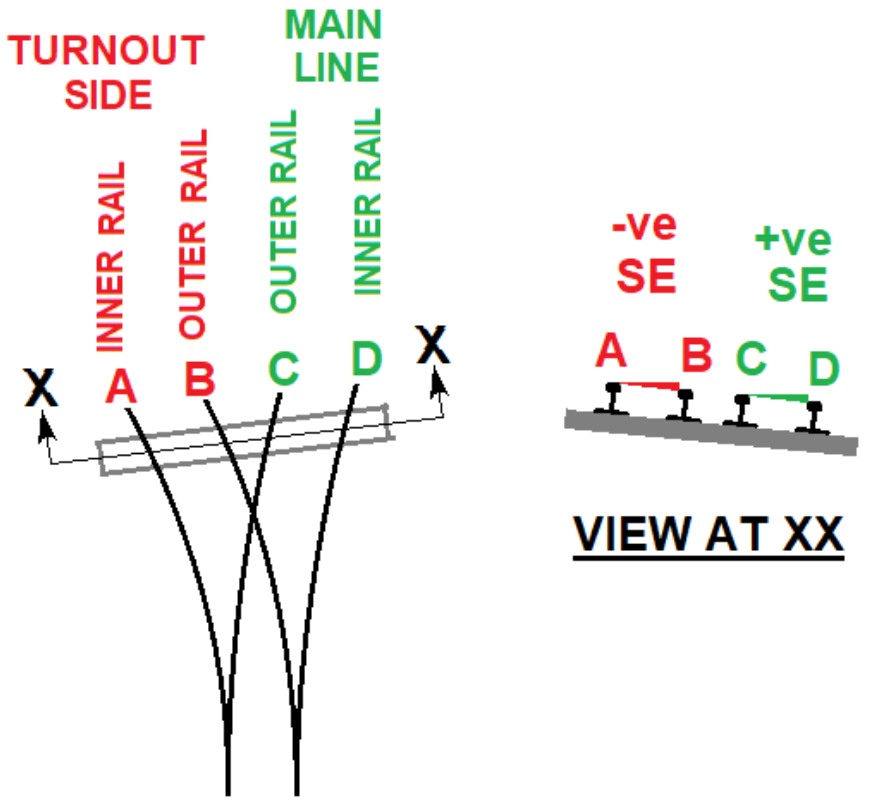


CANT OR SUPERELEVATION

- Superelevation or Cant is the amount by which one rail is raised above the other in a curve.
- It is positive when the outer rail is raised above the inner rail and is negative when the inner rail is raised.

Ref: IRPWM Para 404(1)

- Negative SE occurs in case of contrary flexure turnout.



NEGETIVE SUPERELEVATION IN CONTRARY FLEXURE TURNOUT

EQUILLIBRIUM CANT

- Equilibrium cant corresponding to a speed is the cant that keeps the movement of vehicle on a curved track in equilibrium by exactly balancing the centrifugal force.

- Equilibrium cant in mm cor. To V kmph,

$$e_V = \frac{GV^2}{127\,R}$$

where,

G= Track gauge + rail head width in mm

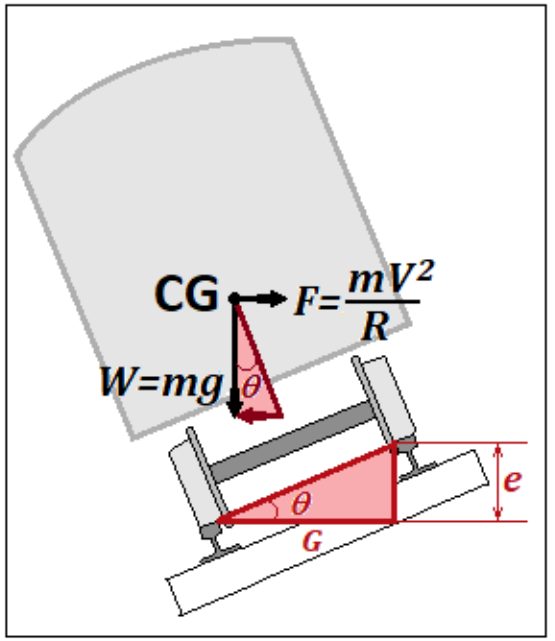
R= Radius in m

Ref: IRPWM Para 406(1)(a)

- Note: Equilibriun cant is also known as theoretical cant.

- Corolary: Equillibrium Speed cor. to given Cant, Ca

$$V_{Ca} = 0.27\sqrt{R.C_a}$$



$$\frac{e}{G} = \tan\theta = \frac{F}{W} = \frac{mV^2}{mgR} = \frac{V^2}{gR}$$

$$e = \frac{GV^2}{gR}$$

If e & G are in mm, V in kmph, R in m and g=9.81 m/s²

$$e = \frac{GV^2}{R} \times \frac{(1000/3600)^2}{9.81 \times 1000}$$

$$e = \frac{G\,V^2}{127\,R}$$

ACTUAL CANT

- The amount of cant to be actually provided (C_a) is to be calculated as the equilibrium cant corresponding to **cant determining equilibrium speed** decided by CE, V_{CE} and nominated for each sub section.

$$C_a = e_{V_{CE}}$$

- The CE decides the cant determining equilibrium speed by considering the speeds of fast & slow trains and the factors that may reduce goods train speed in a subsection.

Note:

This Ca should not exceed the maximum permissible cant in the section.

Cant for each curve should be specified to nearest 5 mm multiple

The specified cant should be indicated on the inside web of inner rail.

Actual cant is specified at the time of original laying of line. Subsequent alteration is done only with CE's approval.

Ref: IRPWM Para 406(1) (b), (c) & (e)

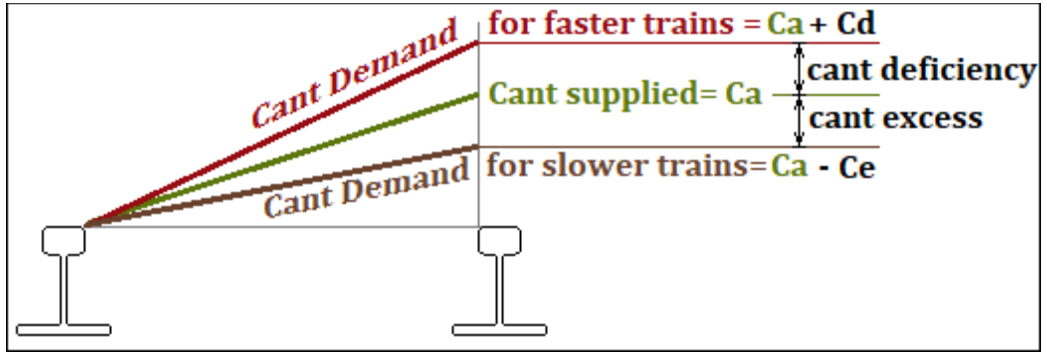
Gauge	Group	Max Perm Cant (mm)	Remarks
BG	**A, B, C**	**165**	For the purpose of locating permanent structures on new constructions and doublings and providing longer transition curve in Gr. A routes having future high speed potential, max **185** mm cant may be assumed.
BG	**D Spl, D, E**	**140**	
MG	All	90	Cant be increased to **100** mm with CE's special permission.
NG (762 mm)		**65**	Can be increased to **75** mm with CE's special permission. For some NG sections, Zonal Railways prescribe special schedules which also should be observed.

CANT DEFICIENCY AND CANT EXCESS

- **Cant Deficiency:** Cant deficiency occurs when a train on curve travels at a speed higher than the equilibrium speed cor to the actual cant. It is the difference between the theoretical cant required for such higher speed and the actual cant provided.

Cant deficiency cor. to V kmph

$$Cd_V = e_V - c_a$$



- **Cant Excess:** Cant excess occurs when a train on curve travels at a speed lower than the equilibrium speed cor to the actual cant. It is the difference between the actual cant and the theoretical cant required for such lower speed.

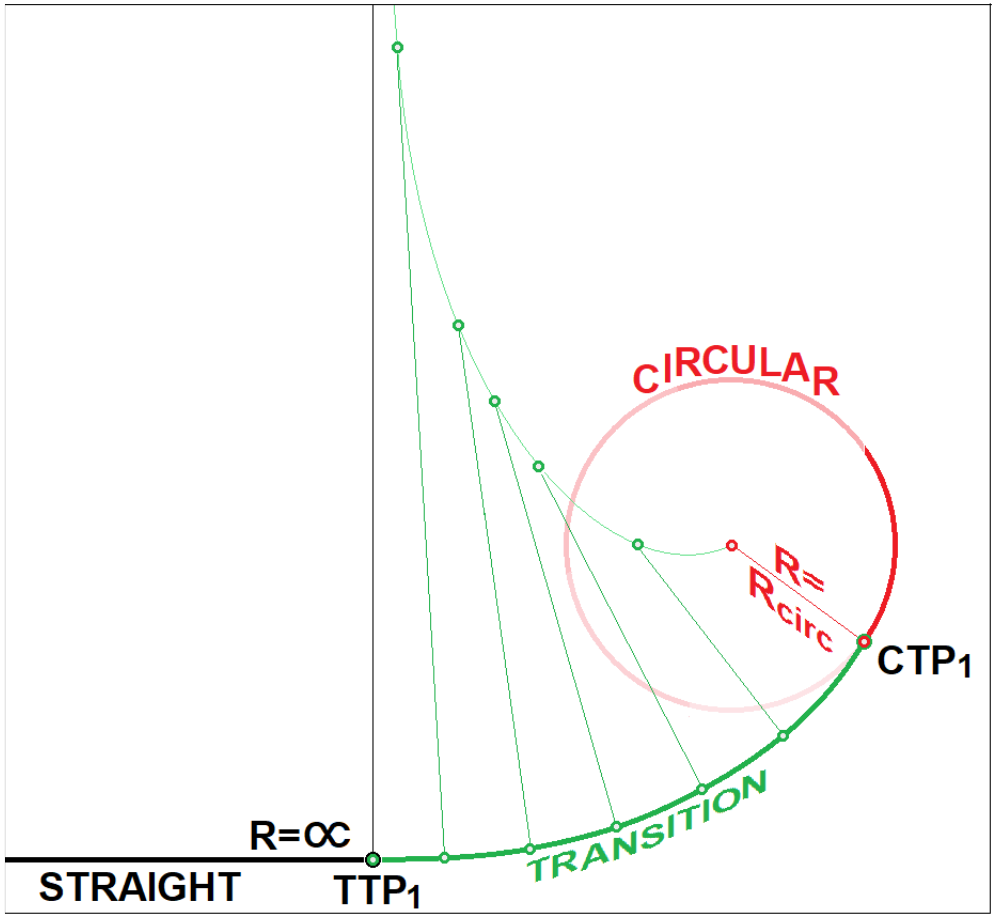
Cant excess cor. to V kmph

$$Ce_V = C_a - e_V$$

Gauge/ Route	Max. Permissible Cant Deficiency		
On BG Routes where track is maintained to C & M-1 Vol-I standards	**100 mm**	**For nominated rolling stock, with permission of PCE**	
For other BG routes	**75 mm**		
On MG routes	50 mm		
On NG (762 mm)	40 mm	For NG sections for which special schedules are prescribed by Zonal Railways, provisions in this schedules should be observed.	

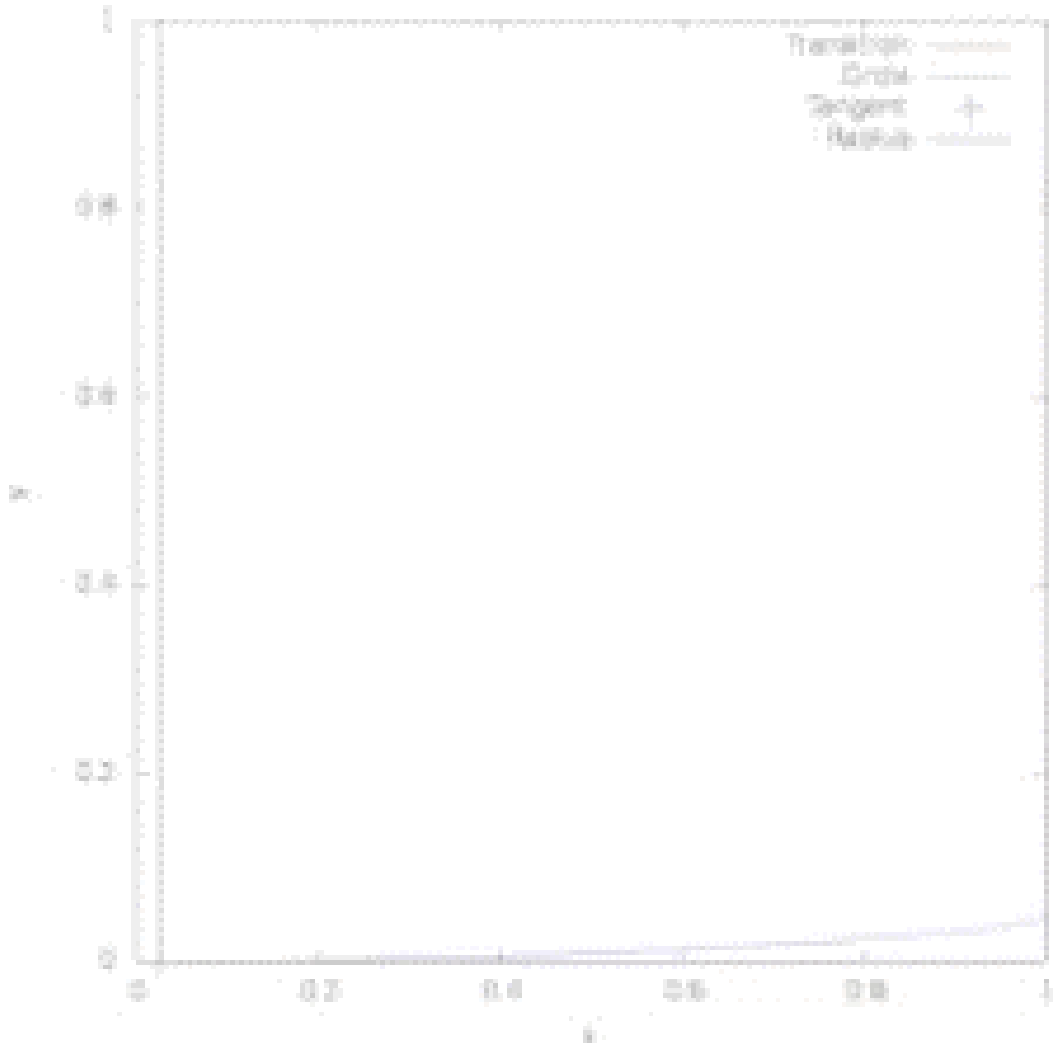
Gauge/ Routes	Max. Permissible Cant Excess	
BG All routes	**75 mm**	for all types of rolling stocks. The cant excess should correspond to the **booked speed of goods train** on a particular station. For sections carrying predominantly goods traffic, cant excess should preferably be kept low to minimise wear on inner rail.
MG All routes	65 mm	

TRANSITION CURVE

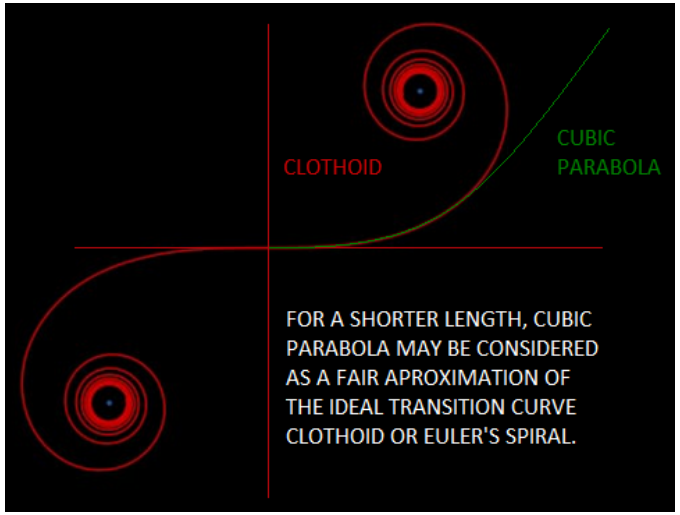


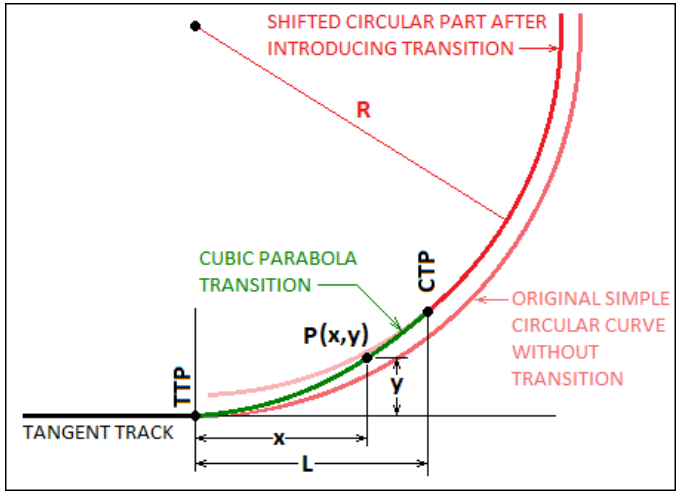
- Transition curve is an easement curve, in which the **change of radius is progressive** throughout its length.
- It affords a **gradual increase of curvature** from zero at the tangent point to the specified radius of circular arc.
- It permits a **gradual increase of superelevation** simultaneously with curvature.
- In IR, transitions are usually provided in a shape of **cubic parabola.**

Ref: IRPWM Para 404(8)


Translogic
Circle
Tangent
Radius
1
0.8
0.6
0.4
0.2
0
y
0
0.2
0.4
0.6
0.8
1
x

Equation of Cubic Parabola Transition Curve





- The equation of the cubic parabola is

$$y = \frac{x^3}{6RL}$$

where

(x,y) denotes the coordinates of a point on the transition commencing from TTP and terminating at CTP.

R is the radius of the circular arc.

L is the length of the transition curve (along the tangent track).

NEXT SESSION TOPICS

- Rate of change of Cant and Rate of change of Cant Deficiency
- Cant gradient
- Length of Transition
- Virtual Transition
- Safe speed on curves
- Maximum Permissible Speed (MPS) of the curve.
- Extra clearance on curves
- Realignment of curve: criteria, stringlining,
- Speed over turnouts on curve
- Compensation for curvature on gradients
- Vertical curve

PLEASE ANSWER THESE QUESTIONS QUICKLY

- The versine on 20 m chord of a curve is 100 mm. Its radius is … m.
- The radius of a 3.5° curve is … m.
- Select a chord length in feet such that the versine of a D° curve on this chord reads D inches.
- Curves can be described by its degree. (T/F)
- The radii of turnout curves during laying should be checked using 6 m chords at an interval of 3.0 m chord (T/F)
- The versine of a 500 m radius turn-in curve is 100 mm. (T/F)
- Versine should be measured on the … rail gauge face at the … point.

THANK YOU